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Then is $DCFG$ or $BDGH$ the required trapezoid. For $BD=GF=b$, $DC=HG=a$, $\angle DKC=\angle BLD=\angle BGC=A$, and $CE=p$. By treating the point m as we did G we get two other trapezoids answering all conditions.

This problem was solved in a similar manner by *COOPER D. SCHMITT*, *A. H. BELL*, *J. SCHEFFER*, *B. F. SINE*, *J. M. COLAW*, *P. S. BERG*, *O. W. ANTHONY*, *E. W. MORRELL*, *J. C. GREGG*, and *H. J. GAERTNER*.

PROBLEMS.

56. Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

The locus of the centers of the isogonal transformations of all the diameters of the circumcircle of any triangle is the nine-points circle. *Brocard.*

57. Proposed by J. OWEN MAHONEY, B. E., Graduate Fellow and Assistant in Mathematics, Vanderbilt University, Nashville, Tennessee.

Show that pairs of points, on a straight line may be so related harmonically that a pair of real points will be harmonic with regard to a pair of imaginary points, and by this means prove that there are an indefinite number of conjugate pairs of imaginary points on a real line.

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CALCULUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

45. Proposed by GEORGE LILLEY, Ph. D., LL. D., Principal of Park School, 394 Hall Street, Portland, Oregon.

A fly starts from a point in the circumference of a table, 3 feet in diameter, and travels uniformly along the diameter to a point in the circumference of the table directly opposite the starting point. The table moves uniformly to the right about a center axis in such manner that it makes one complete revolution while the fly passes over its diameter. Find the absolute path described by the fly and the ratio of rates of movement of the table and the fly.

I. Solution by the PROPOSER.

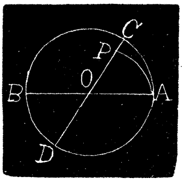
The curve described by the fly is the spiral of Archimedes. Its equation is $r=a\theta$. $S=\int_0^\pi\left(\sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}\right)d\theta, =\frac{a\pi\sqrt{1+\pi^2}}{2}+\frac{a}{2}\log(\pi+\sqrt{1+\pi^2})$.

Hence, $2S$, or the absolute path described by the fly, is 63.994+ inches. If we take the Naperian logarithm of $(\pi+\sqrt{1+\pi^2})$ the result is 69.6+ inches.

The ratio of rates$=\frac{2\pi r}{2r}=\pi$. The ratio of rates in *space*$=\frac{2\pi r}{63.994}=1.76+$.

II. Solution by G. B. M. ZERR, Ph. D., Professor of Mathematics in Texarkana College, Texarkana, Arkansas-Texas; and Professor J. SCHEFFER, A. M., Hagerstown, Maryland.

Let P be the position of the fly when A has moved to C, and let A move m times as fast as P. Let $OA=r$, $OP=\rho$, $\angle COA=\theta$.
Then $mPC=AC$. $\therefore$ $m(r-\rho)=r\theta$.



$\therefore$ $\rho=\frac{r(m-\theta)}{m}=\frac{r(\pi-\theta)}{\pi}$, since $m=\pi$. This is the equation to the fly's path.

$$\therefore\ S=\int_0^{2\pi}\frac{r}{\pi}\sqrt{1+(\pi-\theta)^2}\,d\theta$$

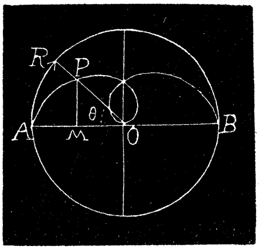
$$=r\sqrt{1+\pi^2}+\frac{r}{\pi}\log(\pi+\sqrt{1+\pi^2}).$$

$$\therefore\ S=\tfrac{3}{2}\left\{\sqrt{1+\pi^2}+\frac{1}{\pi}\log(\pi+\sqrt{1+\pi^2})\right\}=5.835 \text{ feet.}$$

$$\frac{3\pi}{S}=\frac{1885}{1167}=\frac{13}{9}\text{ nearly.}$$

III. Solution by Prof. J. M. BANDY, A. M., Old Trinity College, North Carolina, and J. C. GREGG, Superintendent of City Schools, Brazil, Indiana.

Let $(\rho,\ \theta)$ denote the co-ordinates of P, and since AR and RP are in a constant ratio, ρ and θ are in the same ratio, which denote by c.



Hence, $\theta=-\rho c$ [Archimedean spiral]......(1).
By theory of curves,

$$S=\int\left(\rho^2+\frac{d\rho^2}{d\theta^2}\right)^{\frac{1}{2}}d\theta\ \ldots\ldots\ldots\ldots\ldots\ldots(2).$$

From (1), $\frac{d\rho^2}{d\theta^2}=\frac{1}{c^2}$, and $\rho=\frac{\theta^2}{c^2}$. Substituting these values in (2), $S=\frac{1}{c}\int_0^{2\pi}(1+\theta^2)^{\frac{1}{2}}\,d\theta\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$.

Integrating (3) by formula for reducing $p=1/2$,

$$S=\left[\frac{\theta(1+\theta^2)^{\frac{1}{2}}}{2c}\right]_0^{2\pi}+\frac{1}{2c}\log\left[\theta+\sqrt{1+\theta^2}\right]_0^{2\pi}\ \ldots\ldots\ldots\ldots\ldots\ldots(4).$$

But $c=\frac{\text{arc } AR}{RP}=\frac{\text{circumference}}{\text{diameter}}=\frac{\pi}{1}$. Substituting in (4), and reducing, $S=6.4533+$ feet.

The movement of the fly in its path is the resultant of the motion of the fly along the diameter and the motion of the table to the right about its axis. The rate of motion of the fly in its path is variable, and is measured at any instant by the measuring circle given by any particular value of ρ. So that the ratio of the motion of the table to that of the fly can be found for any particular value of ρ.

Also solved by *O. W. ANTHONY* and *E. L. SHERWOOD*. Prof. Sherwood's solution will be published under problem No. 50.

46. Proposed by H. C. WHITAKER, M. E., Sc. D., Professor of Mathematics, Manual Training School, Philadelphia, Pennsylvania.

There are four points, A, B, C, and D in space. Point D remains fixed with its co-ordinates (1, 2, 2) feet. At a given time A is at (2, 3, 4) feet, is moving in a straight line at the rate of 3 feet per minute, and has passed through (5, 9, 10) feet; B is at (1, 4, 2) feet, moves in a straight line at the rate of 7 feet per minute, and will pass through (—2, 2, 8) feet; C is at the origin and moves along the axis of X in the direction of x positive at the rate of 6 feet per minute.

The motion of the points being continuous before and after the given time, required the times when the volume of the tetrahedron whose edges are the lines joining these points will be 108 cubic inches.

Solution by the PROPOSER.

The length of a base edge [from (x_1, y_1, z_1) to (x_2, y_2, z_2)] is well known to be

$$\sqrt{\begin{vmatrix} x_1 & 1 \\ x_2 & 1 \end{vmatrix}^2+\begin{vmatrix} y_1 & 1 \\ y_2 & 1 \end{vmatrix}^2+\begin{vmatrix} z_1 & 1 \\ z_2 & 1 \end{vmatrix}^2}$$

Finding the distance from $(x_3y_3z_3)$ to this edge, multiplying this distance by the length of the edge just given, the area of the base is

$$\frac{1}{\underline{|2}}\sqrt{\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2+\begin{vmatrix} x_1 & z_1 & 1 \\ x_2 & z_2 & 1 \\ x_3 & z_3 & 1 \end{vmatrix}^2+\begin{vmatrix} y_1 & z_1 & 1 \\ y_2 & z_2 & 1 \\ y_3 & z_3 & 1 \end{vmatrix}^2}$$

Finding the distance from $(x_4y_4z_4)$ to this base, multiply this distance by the area of the base just given, the volume of the tetrahedron is found to be *

$$\frac{1}{\underline{|3}}\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \end{vmatrix}$$

[*The extension of each of these values in n dimentioned co-ordinates is obvious, as is also the sur-solidity of a figure in four dimensioned space bounded by five tetrahedra; and so on.]